

Project 3: Inverse Functions

Learning Objectives

- Define one-to-one and inverse functions
- Explain the horizontal line test and how domain and range are related for inverse functions
- Graph functions and their inverses
- Verify that two functions are inverses of each other
- Determine the inverse of a function given its equation

Assignment Description

The purpose of this assignment is to give you a deeper understanding of inverses of functions. This is an important concept in Chapter 6 as exponential functions and logarithmic functions are inverses of each other, and so you will use one to “undo” the other when solving equations.

As before, feel free to ask questions while doing the project, but don’t wait until the last minute to ask because I’m not up at midnight!

Directions

Let’s start with an example. Suppose we’re told if we take a number, subtract 3, cube it, and divide by 8 we get 1. To figure out the original number, do the opposite of each action:

$$1 * 8 = 8$$

$$\sqrt[3]{8} = 2$$

$$2 + 3 = 5.$$

So 5 is the original number. Now suppose we need to write the original instructions as a function. Let x be the original number, then apply each operation in order. Subtract, cube, and then divide:

$$f(x) = \frac{(x - 3)^3}{8}.$$

When we did the opposite of each operation to find the original number, we found the inverse of the function:

$$f^{-1}(x) = \sqrt[3]{8x} + 3.$$

This is why when you’re finding the inverse of a function you solve for x (which is essentially doing the opposite operations) and then swap the x and y . (Of course, you can swap the x and y and then solve for y -- the order doesn’t matter – but I stated it as I did to align with the example.)

Now you try:

1. Suppose you’re told you have some number, and if you take that number, add 4, take the square root, and subtract 3 you get 1. What is the original number?
- 2.

- a. Write the original operations as a function, $f(x)$.
 - b. What is the domain of $f(x)$?
 - c. Graph $f(x)$ using transformations (section 3.5).
 - d. What is the range of $f(x)$?
3.
 - a. Write $f^{-1}(x)$ (the operations you performed to find the original number).
 - b. In order for $f^{-1}(x)$ to be the inverse of $f(x)$, the domain of $f^{-1}(x)$ needs to be restricted. What is the domain of $f^{-1}(x)$? (See Example 10 on page 418 of the text for more information about domain restriction.)
 - c. Why do you need to restrict the domain of $f^{-1}(x)$? [Hint: think about the horizontal line test and reflection about the line $y=x$.]
 - d. Graph $f^{-1}(x)$ on the same graph from 2b. Be sure to label each graph.
 - e. What is the range of $f^{-1}(x)$?
4. What relationship exists between the domain and range of $f(x)$ and the domain and range of $f^{-1}(x)$?
5. Show that $f(x)$ and $f^{-1}(x)$ are indeed inverses of each other by finding $f^{-1}(f(x))$ and $f(f^{-1}(x))$. Show your work for full credit.

You can write this out by hand and submit a .jpg on Blackboard or submit a Word or similar document. Please do not put your answers on Blackboard directly as it's difficult to read math statements that way.

Assessment

This is project 4 of 5 and is worth 26 points. Your grade for the project will be the percentage of the points you earned out of 26. You will be given credit for correct calculations as well as explaining concepts in a logical manner.

Rubric

Each question is worth 2 points, except for #5, which is worth 4 points. For #5, you earn 2 points for each correct composite function; however, if you have the correct answer but don't show your work you only receive 3 points.

Materials and Technology Needed

- Section 6.2 of your book
- Module 3 SLV 6.1, 6.2 video.